BACKPAPER: ALGEBRA I

Date: **January 2, 2018**
A **ring** would mean a **commutative ring with identity**.

(1) (8+8=16 points) Mention all the options which are correct. No justification needed.
  (a) The composition factors of a finite nontrivial group are:
    (i) cyclic groups
    (ii) simple groups
    (iii) Alternating groups
    (iv) finite groups
  (b) Let G be a finite group, H be a subgroup and P a Sylow p-subgroup of G for a prime number p. Then
    (i) $H \cap P$ is a Sylow p-subgroup of H.
    (ii) $H \cap P$ is a Sylow p-subgroup of H if H is a normal subgroup.
    (iii) None of the above.

(2) (14 points) Prove or Disprove. There is a unique group of order 77 up to isomorphism.

(3) (15 points) Let p be a prime and $|G| = p^n$ for some $n > 0$. Show that the center of G is non trivial.

(4) (5+15=20 points) Define local ring. Show that the power series ring $R[[X]]$ is a local ring if R is a local ring.

(5) (5+15=20 points) Let R be a ring. Define noetherian R-module. Let $0 \to A \to B \to C \to 0$ be a short exact sequence of R-modules. If A and C are noetherian module show that B is noetherian R-module.

(6) (15 points) Let R be a subring of $\mathbb{R}[t]$ strictly containing $\mathbb{R}$. Show that $\mathbb{R}[t]$ as an R-module is finitely generated.